

Quasinormal modes of the electrically charged dilaton black hole

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Abstract

We sketch the results of calculations of the quasinormal frequencies of the electrically charged dilaton black hole. At the earlier phase of evaporation (Q is less than $0.7-0.8M$), the dilaton black hole "rings" with the complex frequencies which differ negligibly from those of the Reissner-Nordström black hole. The spectrum of the frequencies weakly depends upon the dilaton coupling.

When perturbing a black hole there appear damped oscillations with complex frequencies which are the eigenvalues of the wave equation satisfying the appropriate boundary conditions. Usually these are the requirements of purely outgoing waves near infinity and purely ingoing near the horizon. Both the complex part of the QN frequency (inversely proportional to the damping time) and the real one (representing the actual frequency of the oscillation) are independent of the initial perturbations and thereby characterize a black hole itself. The quasinormal spectrum of the neutron stars and black holes is intensively investigated now, since it is in the suggested range of the gravitational wave detectors (LIGO, VIRGO, GEO600, SPHERE) which are under construction.

Frequencies of the quasinormal modes of the electrically charged BH were calculated in several papers long time ago (see [1] and references therein). Yet, on various ground, the main of which are suggestions of supergravity, one ascribes to a black hole a scalar (dilaton) field. The latter changes properties of a black hole, and it seems interesting to find out what will happen to the quasinormal spectrum when adding a dilaton charge to a black hole. Certainly, one should expect that for small charges of the electromagnetic and dilaton fields the spectrum will not differ seemingly from that of the R-N black hole, and, even though the black holes we see today, apparently, do not have large electric charge, the problem is of interest, since in charged environment electromagnetic waves will lead to gravitational ones thereby giving a simple model for studying of the conversing of gravitational energy into electromagnetic one and vice versa.

We shall consider theories including coupling gravitational, electromagnetic and scalar fields with the action:

$$S = \int d^4x \sqrt{-g} (R - 2(\nabla\Phi)^2 + e^{-2a\Phi} F^2) \quad (1)$$

A static spherically symmetric solution of the equations following from this action represents, in particular, electrically charged dilaton black hole with the metric in the form:

$$ds^2 = \lambda^2 dt^2 - \lambda^{-2} dr^2 - R^2 d\theta^2 - R^2 \sin^2 \theta d\varphi^2 \quad (2)$$

where

$$\lambda^2 = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}}, \quad R^2 = r^2 \left(1 - \frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}}, \quad (3)$$

and

$$2M = r_+ + \left(\frac{1-a^2}{1+a^2}\right) r_-, \quad Q^2 = \frac{r_- r_+}{1+a^2}. \quad (4)$$

Here the dilaton and electromagnetic fields are given by the formulas:

$$e^{2a\Phi} = \left(1 - \frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}}, \quad F_{tr} = \frac{e^{2a\Phi} Q}{R^2}, \quad (5)$$

where a is a non-negative dimensionless value representing coupling. The case $a = 0$ corresponds to the classical Reissner-Nordström metric, the case $a = 1$ is suggested by the low energy limit of the superstring theory. The uniqueness of static, asymptotically flat spacetimes with non-degenerate black holes in Einstein-Maxwell-dilaton theory was proved recently when either $a = 1$, or a is arbitrary but one of the fields, electric or magnetic, is vanishing [2].

The perturbations obey the wave equations:

$$\left(\frac{d^2}{dr_*^2} + \sigma^2\right) Z_{1,2} = U_{1,2} Z_{1,2} \quad (6)$$

governed in axial case by the effective potentials:

$$U_{1,2} = \frac{1}{2} \left(V_1 + V_2 \pm \sqrt{(V_1 - V_2)^2 + 4V_{12}^2} \right), \quad (7)$$

where

$$V_1 = a^2(\Phi_{,r*})^2 - a\Phi_{,r*r*} + (\mu^2 + 2)\lambda^2 R^{-2} + 4Q^2 \lambda^2 e^{2a\Phi} R^{-4}, \quad (8)$$

$$V_2 = 2R^{-2}(R_{,r*})^2 - R^{-1}R_{,r*r*} + \mu^2 \lambda^2 R^{-2}, \quad V_{12} = -2Q\mu e^{a\Phi} \lambda^2 R^{-3}. \quad (9)$$

Here $dr = \lambda^2 dr^*$ and $\mu^2 = l(l+1) - 2$, where l is the angular harmonic index. The wave equation governed by the first (second) potential at $Q = 0$ corresponds to the electromagnetic (gravitational) perturbations, and at $Q \neq 0$ each QN-mode will be connected with emission of both electromagnetic and gravitational radiation. These potentials were obtained in the work [3], where a complete analysis of the perturbations of the dilaton black hole was done. We have calculated the complex quasinormal frequencies for the above class of black holes.

Finding of the quasinormal frequencies for black holes with reasonable accuracy is not such a time consuming process any more: one can use the third order WKB formula (1.5) of the paper [4], and then, in order to improve accuracy, use the obtained values of the frequencies as initial guesses in the Chandrasekhar-Detweiler numerical method [5]. For a recent review of the methods see [6]. In the present paper we were restricted by the lower overtone modes, as those dominating in a signal.

First, we observed that in the axial case the complex QN-frequencies corresponding to the gravitational perturbations almost do not depend on the value of the coupling a of the dilaton field in the wide range from $a = 0$ up to $a \sim 100$, unless the electric charge (in mass units) is too large ($Q \simeq 0.7 - 0.8M$). We illustrate this for fundamental modes, i.e. for modes with $l = 1$, $n = 0$, where n is the overtone number in Tab.1. This dependence on a is still weak for the electromagnetic perturbations.

Tab.1 The fundamental quasi-normal frequencies corresponding to the gravitational perturbations, axial case.

$Q = 0.2$			$Q = 0.9$		
a	$Re(\omega)$	$-Im(\omega)$	a	$Re(\omega)$	$-Im(\omega)$
0	0.11252	0.10040	0	0.13275	0.09980
2	0.11252	0.10040	2	0.13190	0.10098
4	0.11251	0.10041	4	0.12936	0.10294
8	0.11249	0.10043	8	0.12611	0.10431
16	0.11242	0.10044	16		
100	0.11198	0.10047	100		

Certainly, the more a , the less charge, at which the discrepancy with the Reissner-Nordström QN-frequencies becomes considerable, but at the earlier stage of evaporation the dilaton black hole "rings" with the frequencies which are negligibly different from the R-N frequencies, no matter the value of the coupling constant from the above region.

From the Fig.1-Fig.3 we see that as for the classical R-N solution, in the $a = 1$ case, which is of our main interest, the real frequencies increase with increasing of the electric charge, and the inverse damping times are increasing also up to some maximal value at a large charge and then falling off. However this picture takes place for the dilaton black hole with some kind of "retarding": the real part of the quasi-normal frequency is less than that of the Reissner-Nordström black hole with the same charge, and, the corresponding inverse damping time is greater than that of the Reissner-Nordström. This tendency can easily be explained if taking into consideration that the dilaton field contributes an extra attractive field which partly compensate the effect from the increased electric charge, thereby inducing the above changes in the quasinormal spectrum.

Somewhat surprisingly, the real part of the quasinormal modes corresponding to the gravitational perturbations of the $a = 1$ dilaton black hole shows no more than 0.6 percent "relative deviation" from R-N for any charge $0 < Q \leq 0.99M$, i.e.

$$\text{Re } \omega_{\text{dilatonBH}} = (1 - \epsilon) \text{Re } \omega_{\text{R-NBH}}, \quad (10)$$

where

$$\max \epsilon \approx 0.006 \text{Re } \omega_{\text{R-NBH}}, \quad \epsilon > 0. \quad (11)$$

Nevertheless, the corresponding damping times differ seemingly when the charge is of order $0.7 - 0.8M$.

For large l , from the first order WKB method [9] we obtain approximate formula for the slowest damped mode:

$$\text{Re } \omega \approx \frac{1}{r_0} (r_0 - 2M)^{\frac{1}{2}} \left(r_0 - \frac{Q^2}{M} \right)^{-\frac{1}{2}} \left(l + \frac{1}{2} \right) \quad (12)$$

$$\begin{aligned} \text{Im}\omega \approx & -\frac{1}{2r_0^2(Mr_0 - Q^2)}(30M^4r_0^2 + Q^4r_0^2 - 3MQ^2r_0(3Q^2 + r_0^2) - \\ & 2M^3(21Q^2r_0 + 10r_0^3) + M^2(16Q^4 + 25Q^2r_0^2 + 3r_0^4))^{\frac{1}{2}} \end{aligned} \quad (13)$$

where r_0 is the value of r where the black hole potential attains its maximum;

$$4Mr_0 \approx 6M^2 + Q^2 + \sqrt{36M^4 - 20M^2Q^2 + Q^4}. \quad (14)$$

It is clear that, in full analogy with the Reissner-Nordström black hole behavior, $\text{Im}\omega$ of the $a = 1$ dilaton black hole in the large- l limit tends to a negative constant, while $\text{Re}\omega$ increases linearly with l . From Fig.4 one can see that at large l , ω shows the same behavior with changing Q as for small one.

As is known when perturbing the R-N black hole, the QN-modes induced by the axial and polar perturbations are identical [1]. In addition, the R-N QN-modes corresponding to the gravitational and electromagnetic perturbations coincide in the extremal limit [7], supporting the fact that only the extremal black hole preserves supersymmetry [8]. Both these symmetries are broken in the case of the electrically charged dilaton black hole.

¹ We expect that is the extremal dilaton black hole with *both* electric and magnetic charges, being N=4 supersymmetric when embedded in N=4 supergravity [10], which must respond in the same manner on gravitational and electromagnetic perturbations, and to check it is the point of our future investigation.

It worthwhile mentioning, that in the axial case, the dilaton field itself does not suffer from perturbations unlike the polar one. Yet the polar perturbations of the dilaton black hole are governed by a very cumbersome potentials, and we have not found a better way than, following the paper [3], to consider the spectator scalar field propagating in the black hole background as a qualitative model for perturbations. Consequently, the WKB accuracy was sufficient under such qualitative consideration. It proved out that the above general properties of the axial modes, are valid in the considered case as well and, apparently, spread on polar modes.

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¹Even though in the $a = 1$ case we could not compute the QN-frequencies accurately enough when approaching too close to the extremal limit with the unmodified Chandrasekhar-Detweiler or WKB methods due to the broadening of the effective potentials [3], the values of the quasinormal frequencies we obtained for $Q = 1.41M$ do not leave any hope that the frequencies for gravitational and electromagnetic perturbations will coincide in the extremal limit.

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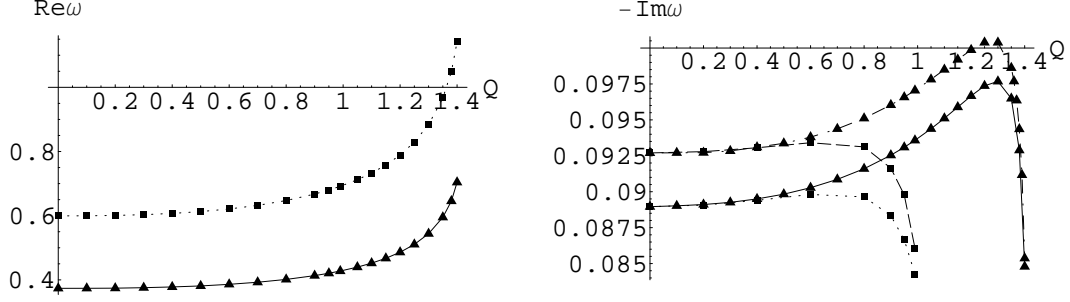


Figure 1: Real and imaginary parts of ω , $l = 2, 3$, for axial gravitational perturbations of the $a = 1$ dilaton black hole and R-N black hole. For $0 < Q \leq 0.99M$, $\text{Re}\omega_{\text{dilatonBH}} = (1 - \epsilon) \text{Re}\omega_{\text{R-NBH}}$, where $\max \epsilon \approx 0.006 \text{Re}\omega_{\text{R-NBH}}$.

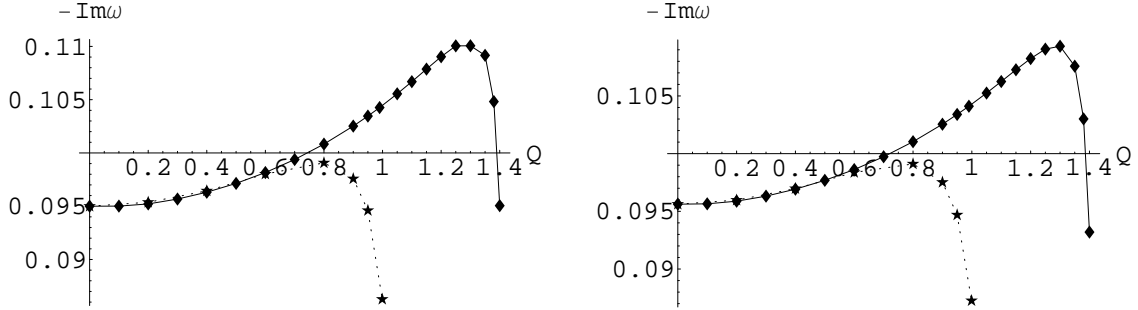


Figure 2: Imaginary part of ω for axial electro-magnetic perturbations of the $a = 1$ dilaton black hole and R-N black hole; $l = 2$ and $l = 3$.

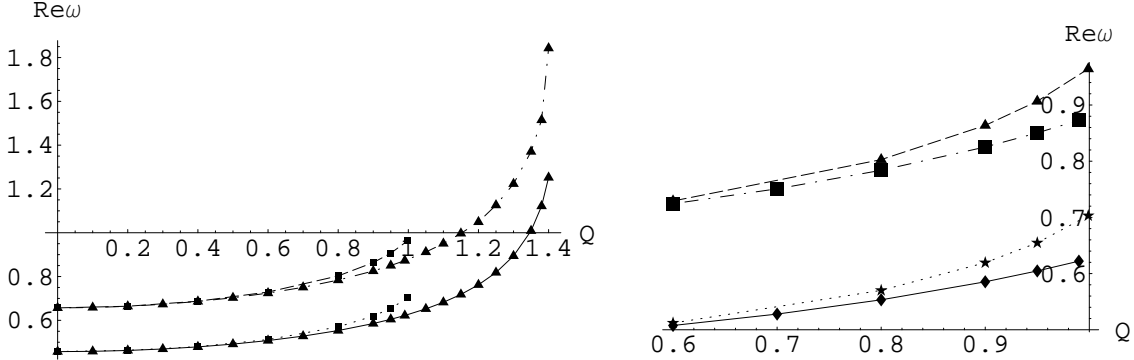


Figure 3: Real part of ω for axial electro-magnetic perturbations ($l = 2, 3$) for $a = 1$ dilaton black hole and for R-N one. Enlarged region of the figure shows when the difference between the R-N QN-modes and those of its dilaton analog can not be ignored.

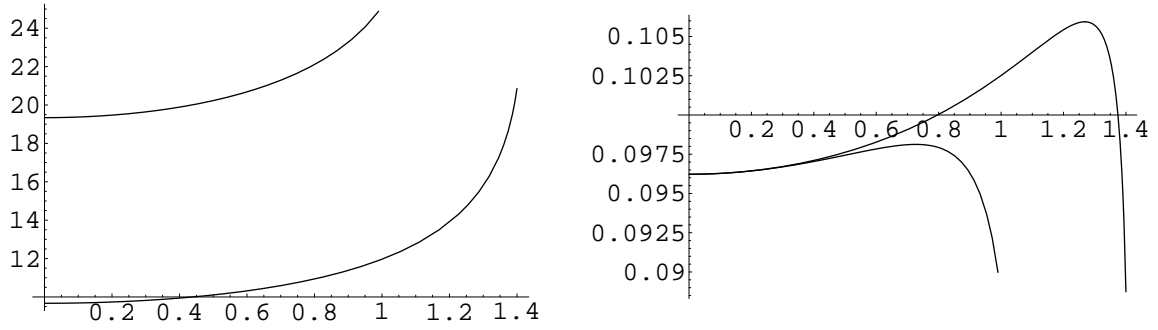


Figure 4: Real and imaginary parts of ω for large l as an approximate function of Q for $a = 1$ dilaton black hole (by the formulas (12-14)) and for R-N (by the formulas (4-5) of the work [11]) ($M = 1$, $l = 100$).